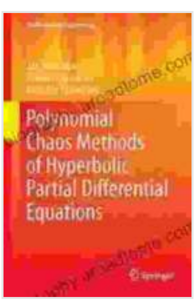
Polynomial Chaos Methods For Hyperbolic Partial Differential Equations: Unraveling Complex Phenomena

Partial differential equations (PDEs) are powerful mathematical tools that model a wide range of physical phenomena, from fluid flow to heat transfer. Hyperbolic PDEs are a particularly important class of PDEs that describe wave propagation and other transient phenomena. However, solving hyperbolic PDEs can be challenging, especially when the equations are nonlinear or the input data is uncertain.

Polynomial chaos methods (PCMs) are a powerful class of techniques for solving PDEs with uncertain inputs. PCMs represent the solution to a PDE as a polynomial chaos expansion, which is a series of terms that are orthogonal with respect to a measure of uncertainty. This representation allows for efficient and accurate uncertainty quantification, even for complex and nonlinear PDEs.



Polynomial Chaos Methods for Hyperbolic Partial Differential Equations: Numerical Techniques for Fluid Dynamics Problems in the Presence of Uncertainties (Mathematical Engineering)

★★★★★ 5 out of 5

Language	: English
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Screen Reader	: Supported
Enhanced typesetting	: Enabled
Print length	: 379 pages

Polynomial Chaos Expansion

The polynomial chaos expansion of a random variable X is given by:

$$X = \sum_{k=0}^\infty c_k \phi_k(\xi)$$

where c_k are the polynomial chaos coefficients, ϕ_k are the polynomial chaos basis functions, and ξ is a random variable that is uniformly distributed on the interval $[-1, 1]$.

The polynomial chaos basis functions are orthogonal with respect to the measure of uncertainty. This means that:

$$\int_{-1}^1 \phi_i(\xi) \phi_j(\xi) d\xi = \delta_{ij}$$

where δ_{ij} is the Kronecker delta.

Solving Hyperbolic PDEs with PCMs

PCMs can be used to solve hyperbolic PDEs by representing the solution as a polynomial chaos expansion. The coefficients of the expansion are then determined by solving a system of ordinary differential equations. This approach can be used to solve both linear and nonlinear hyperbolic PDEs.

The main advantage of using PCMs to solve hyperbolic PDEs is that it allows for efficient and accurate uncertainty quantification. This is because the polynomial chaos expansion provides a complete representation of the solution, including all of the moments and statistical properties.

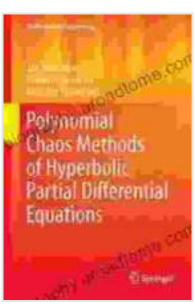
Applications of PCMs

PCMs have a wide range of applications in science and engineering, including:

- Fluid dynamics
- Structural engineering
- Heat transfer
- Combustion
- Wave propagation

In fluid dynamics, PCMs can be used to model turbulent flows, shock waves, and other complex phenomena. In structural engineering, PCMs can be used to analyze the safety and reliability of structures under uncertain loads. In heat transfer, PCMs can be used to model the flow of heat in complex systems.

Polynomial chaos methods are a powerful tool for solving hyperbolic partial differential equations. These methods allow for efficient and accurate uncertainty quantification, making them a valuable tool for a wide range of applications in science and engineering.



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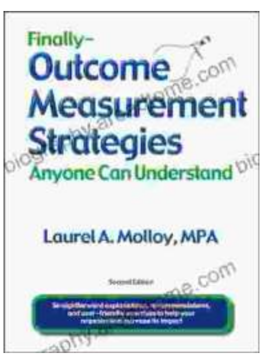

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